

# Supplementary Information: Underpinning hybridization intuition for complex nanoantennas by magnetoelectric quadrupolar polarizability retrieval

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## Quadrupolar fields and $12 \times 12$ $\alpha^S$ -tensor

Here we show that given the choice of casting the rank 2 quadrupole tensor in a vector, and reducing redundancy due to the tensor symmetry, the  $\diamond$  operator is completely determined. Starting from the symmetric traceless quadrupole tensor  $Q$ , the field emitted by a quadrupole in Einstein notation is

$$E_i(\mathbf{r}) = \frac{\omega^2 \mu \mu_0}{3!} Q_{jk} \nabla'_k \mathbf{G}_{ji}(\mathbf{r}, \mathbf{r}'), \quad (1)$$

with  $\omega$  the optical frequency,  $\mu \mu_0$  the permeability of the surrounding medium and  $G$  the  $3 \times 3$  electric Green dyadic. That the tensor  $Q_{ij}$  is symmetric allows a regrouping of terms

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(where we abbreviate  $\partial_{x'} \mathbf{G}_{ij}(\mathbf{r}, \mathbf{r}')$  as  $G_{ijx'}$ )

$$\mathbf{E} = \frac{\omega^2 \mu \mu_0}{3!} \begin{pmatrix} Q_{12}(G_{11y'} + G_{12x'}) + Q_{13}(G_{11z'} + G_{13x'}) + Q_{23}(G_{12z'} + G_{13y'}) + Q_{11}G_{11x'} + Q_{22}G_{12y} + Q_{33}G_{13z'} \\ Q_{12}(G_{21y'} + G_{22x'}) + Q_{13}(G_{21z'} + G_{23x'}) + Q_{23}(G_{22z'} + G_{23y'}) + Q_{11}G_{21x'} + Q_{22}G_{22y} + Q_{33}G_{23z'} \\ Q_{12}(G_{31y'} + G_{32x'}) + Q_{13}(G_{31z'} + G_{33x'}) + Q_{23}(G_{32z'} + G_{33y'}) + Q_{11}G_{31x'} + Q_{22}G_{32y} + Q_{33}G_{33z'} \end{pmatrix} \quad (2)$$

which can be identified as

$$\mathbf{E} = \frac{\omega^2 \mu \mu_0}{3!} (\diamond' \overline{\mathbf{G}}^\top(\mathbf{r}, \mathbf{r}'))^\top \cdot \mathbf{Q}. \quad (3)$$

Thereby defining  $\mathbf{Q}$  as a 6-vector as in the main manuscript fixes the definition of  $\diamond$ . Mutatis mutandi the same reasoning holds if in Eq. (2) one eliminates  $Q_{zz}$  on basis of the traceless nature of the tensor  $Q$ , in which case inserting the definition of  $\hat{\mathbf{Q}}$  fixes  $\hat{\diamond}$ .

## Dolmen $\alpha^S$ -tensor elements

To complement our report in the main manuscript of elements of  $|\alpha^S|$  at one wavelength and as a logarithmic color scale for the silver dolmen, here we report the superpolarizability tensor element amplitude and phase versus wavelength grouped in three figures, i.e. Fig.1,2 and 3.

## Gold disk $\alpha^S$ -tensor elements

To complement our report in the main manuscript of elements of  $|\alpha^S|$  at one wavelength and as a logarithmic color scale for the Au nanodisk on a Si substrate, here we report the superpolarizability tensor element amplitude and phase versus wavelength in Fig.4.

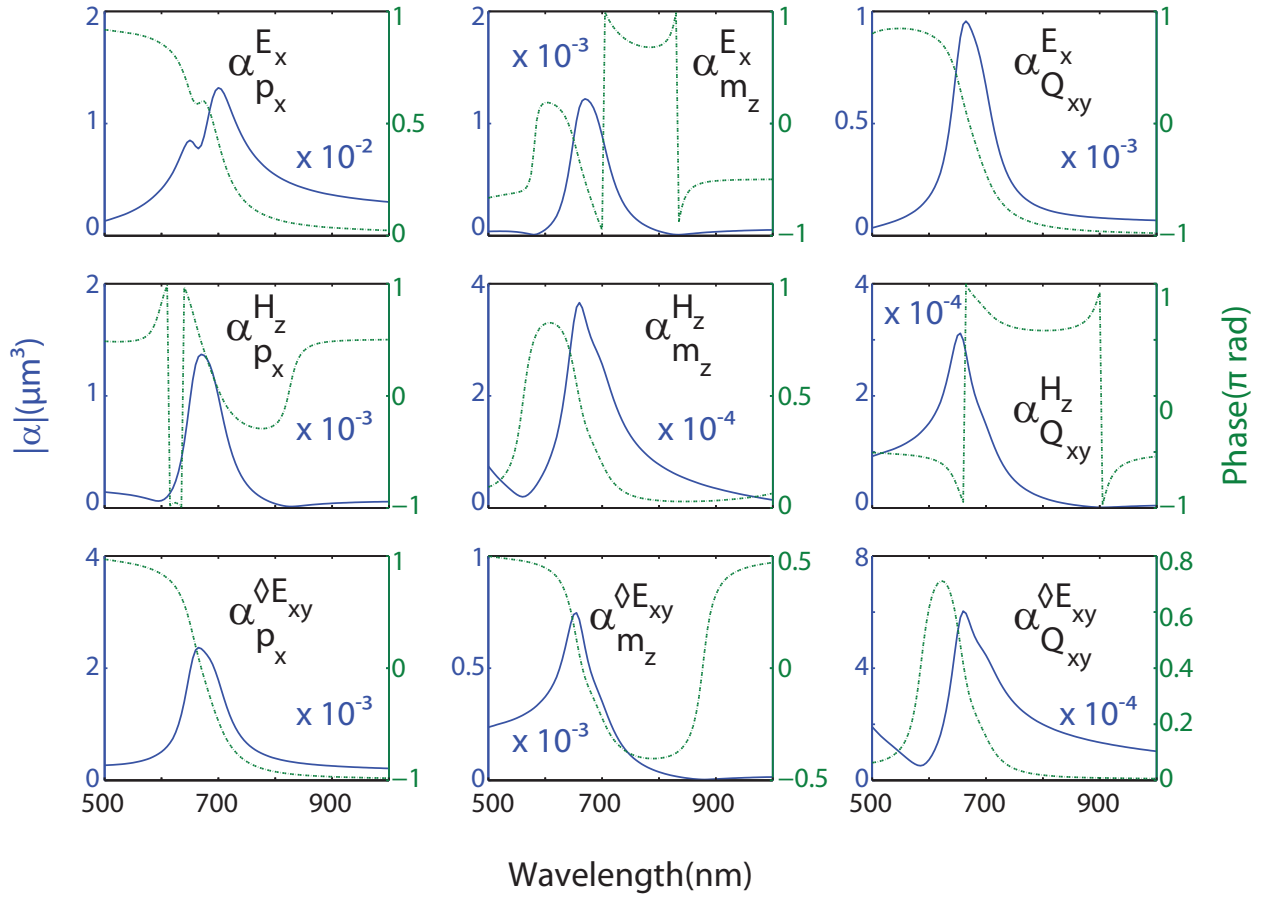


Figure 1: Elements related to  $p_x$ ,  $m_z$  and  $Q_{x,y}$  of the  $\alpha^S$ -tensor of a silver dolmen structure. These elements span the subspace relevant for PIT in the dolmen system. Solid curves show amplitude while dashed curves show phase.

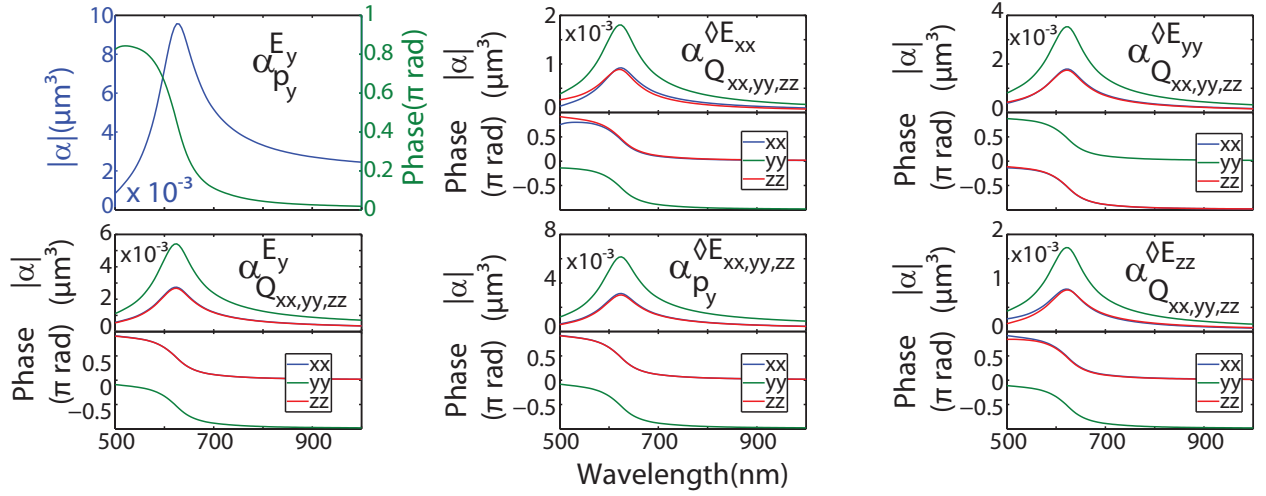


Figure 2: Elements related to  $p_y$  and  $Q_{xx,yy,zz}$  of the  $\alpha^S$ -tensor of a silver dolmen structure. These elements correspond to excitation of the symmetric dimer resonant mode, which has a strong electric dipole character, as well as carrying a linear  $y$ -oriented quadrupole moment.

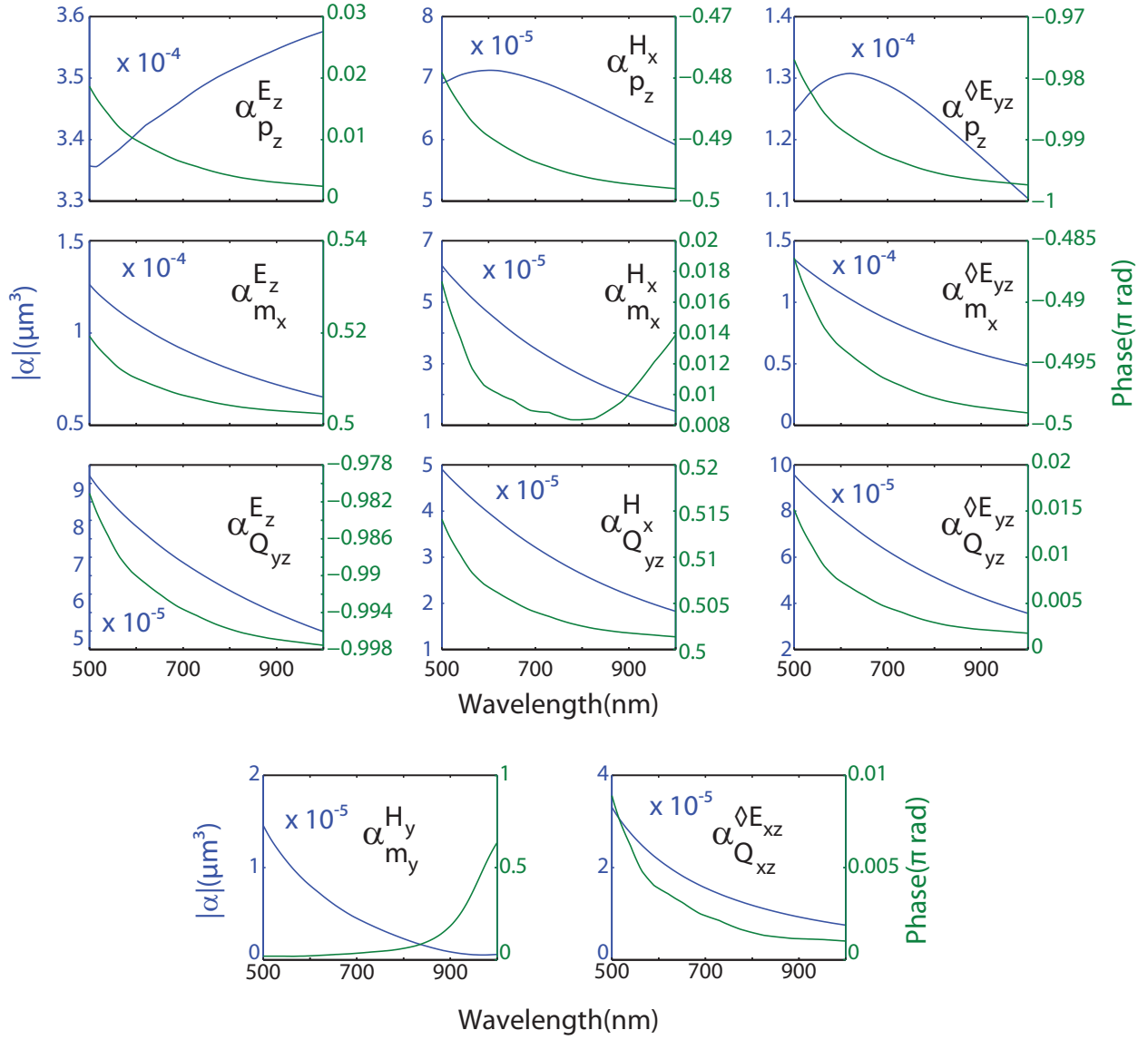


Figure 3: Summary of weaker elements of the  $\alpha^S$ -tensor of the silver dolmen structure, generally corresponding to out-of-plane charge separation.

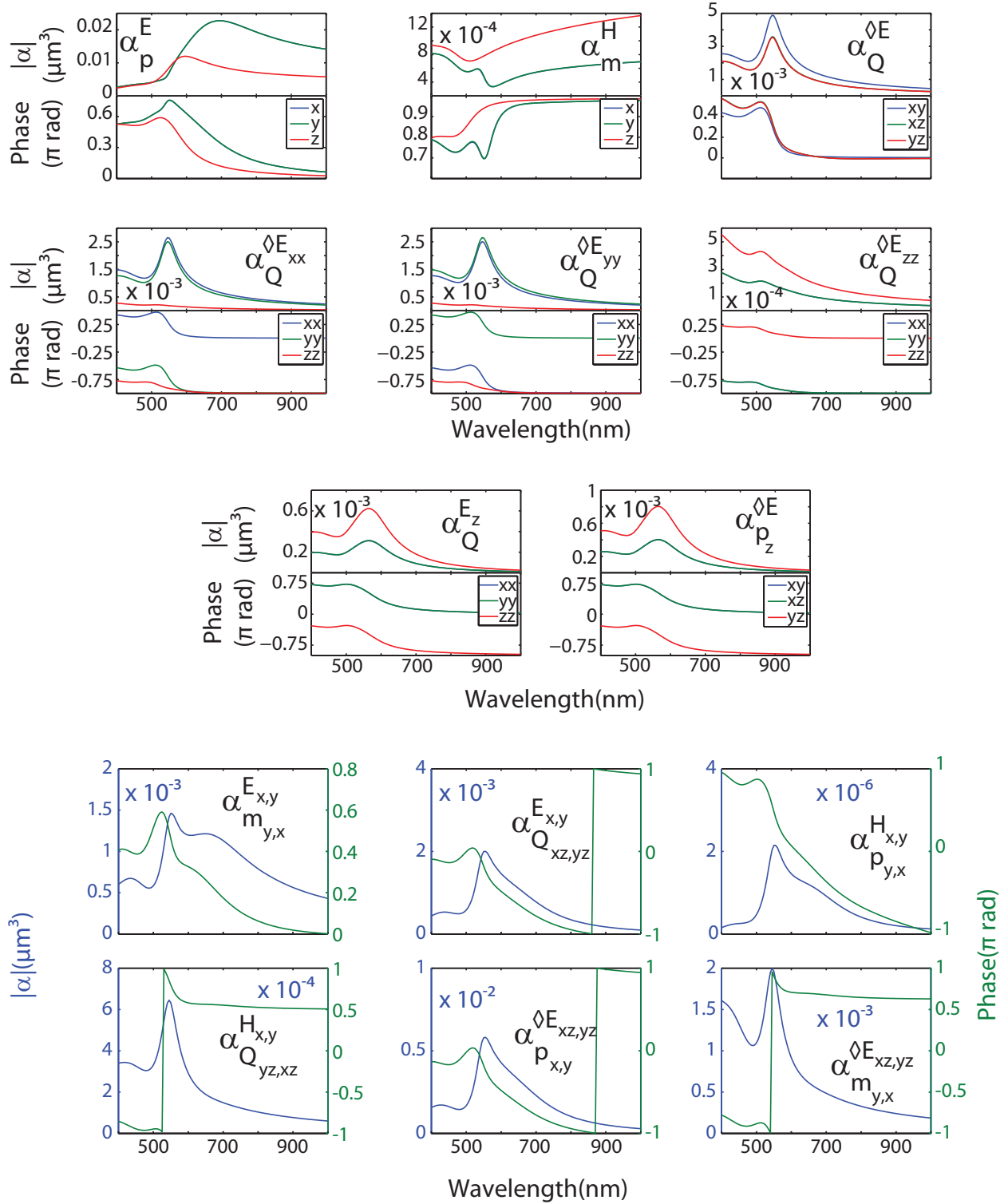


Figure 4: Elements of the  $\alpha^S$ -tensor of a gold cylinder with 180 nm diameter and 80 nm height.