Lecture Nanophotonics
Problem set 1: a feeling for numbers

Handout date:	Monday, March 30 2020
Due date:	Sunday, April 5 2020
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Enrico Fermi was known for his ability to make approximate calculations with little or no actual input data, using dimensional analysis and simple back-of-the-envelope estimates. A famous problem he would pose students is "How many piano tuners are there in Chicago?" Dealing with such "Fermi-questions" is an important skill since in active science, most questions are poorly posed, often a number of variables are not understood, and it would be difficult to find answers exactly. The goal of this exercise is to take a poorly-defined problem, as most real-world problems are, and come up with a workable answer to it. In this problem set we encourage you to look up numbers and equations you need. Explain your assumptions and reasoning steps, and cite your sources.

The way to approach these problems is as follows:

1. what do you want to know?
2. what do you know, and what are the units?
3. which formula do I need?
4. what are reasonable numbers to plug into this formula?

Example: How many piano tuners are there in Amsterdam?
We want to know how many people make a living tuning piano's in the city of Amsterdam. We know that roughly a million people live in larger Amsterdam. We can find out how many pianos need tuning by estimating what percentage of those people own a piano, σ.
We can estimate the number of tunings a tuner can do in a year by estimating how much time you need to tune a piano (T_1, unit is [time worked/tuning]), and how much someone works in a year (T_2, unit is [time worked/year]). By doing dimensional analysis, we know that we need something that has the unit of [tunings/year]. We'll call it N_t, the number of tunings a piano needs in a year.
A somewhat reasonable formula:

$$N = \frac{N_{\mathrm{Ams}}}{\sigma} \times \frac{T_1[\mathrm{time/tuning}]}{T_2[\mathrm{time/year}]} \times N_t[\mathrm{tunings/year}] \tag{1}$$

Where

$$N_{\mathrm{Ams}} = 10^6 \text{ number of people in Amsterdam}$$
$$\sigma = 0.01 \text{ estimate for what fraction of people own a piano}$$
$$T_1 = 2 \text{ hours per tuning, according to [1]}$$
$$T_2 = 8 \text{ hours a day} \times 20 \text{ days a month} \times 12 \text{ months a year} = 1920 \text{ hours/year}$$
$$N_t = 2 \text{ per year, according to [2]}$$

So we find that the number of piano tuners in Amsterdam must be around 21 full-time piano tuners in Amsterdam, which seems pretty reasonable.
You will notice that there are many different ways to come to a reasonable answer.

Protip: you can really help yourself by paying close attention to the units of your variables. This can give you valuable hints about where you need to search next.

Exercises

1. Trapping light for a very long time in an optical resonator is popular business, going from nanophotonics to gravitational wave detection. (a) If you want an optical cavity with a lifetime of one minute, how far apart do you need to place two simple household mirrors? (b) The best optical microcavities in nanophotonics are $\sim$1 micron in size, and resonate for nanoseconds. What is the apparent reflectivity of these cavity mirrors?

2. How small does a cavity mode volume need to be so that the field strength of a single photon [visible light frequency] confined in it approaches the Coulomb field felt by an electron around an atomic nucleus? You can assume that the energy density (unit [J/m^3]) of a light field is $\epsilon_0 |E|^2$, where ϵ_0 is the vacuum permittivity, and E is the electric field strength.

3. Solar cells are said to be easily capable of supplying all of the Netherlands' energy.

   (a) How many km^2 of solar cells would we need to power the Netherlands?

   (b) How much would this cost?

   (c) Can we fit this on the roofs?

4. One of the 2014 Nobel prizes (Moerner, Hell and Betzig) was for being able to see single molecules with an optical microscope using "fluorescence". In fact, you could rather say it was for *not* seeing a background of "dark" molecules that form the host material containing the fluorophore. How many molecules fit inside the focus of an optical microscope objective? Given that a fluorescent molecule emits at most one photon every nanosecond, how much dimmer then a light-bulb is a single molecule?

5. Your phone is able to capture a good image of a full moon without too much motion blur, on its CCD/CMOS camera. Use this to estimate the sensitivity of a single pixel, where the sensitivity is the optical energy to get an appreciable signal-to-noise ratio. Assume the standard light intensity of the full moon and commercial phone camera dimensions.

Sources
[1] https://modernpianoboston.com/faq/2016/4/20/how-long-does-it-take-to-tune-a-piano
[2] http://www.concertpitchpiano.com/HowOftenAndWhen.html