Nanophotonics

Nanoscale:	10^{-9} meter
Photonics:	*science of controlling propagation, absorption & emission of light (beyond mirrors & lenses)*

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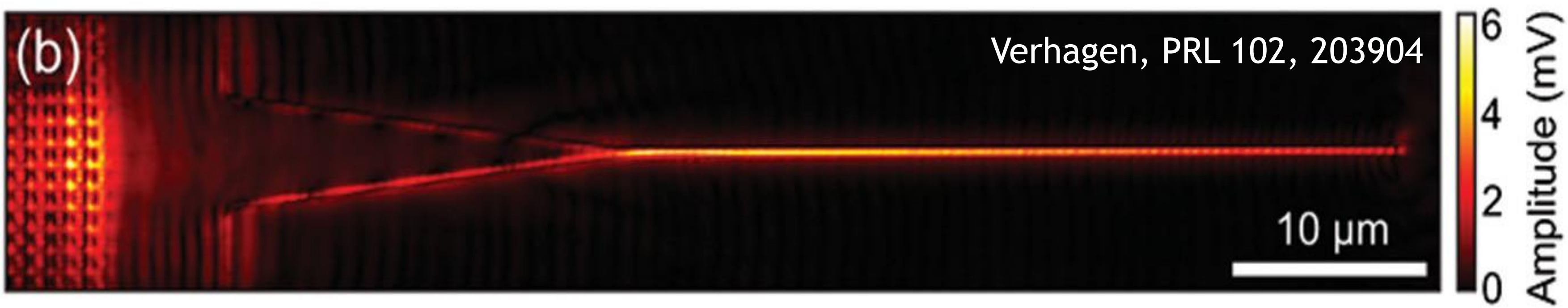
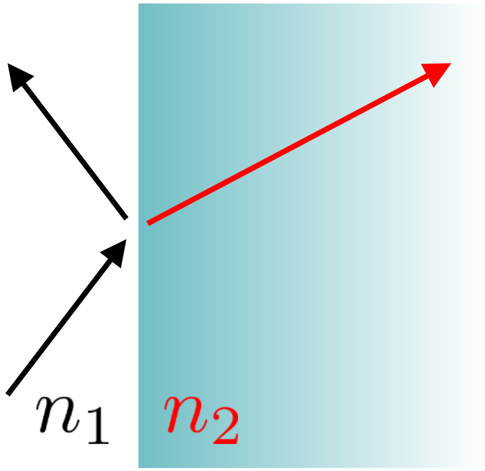
physics of functional complex matter

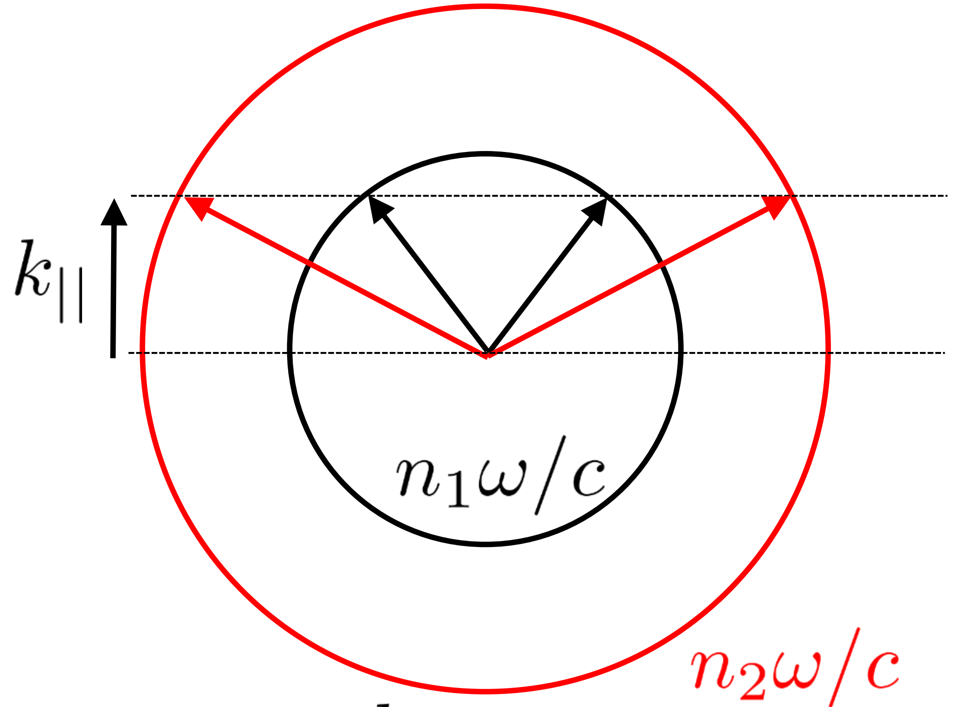
Today

- Revisiting Fresnel – a single interface
- Refractive index of metals & dielectrics
- Trapping light in waveguides and circuits



Snell's law



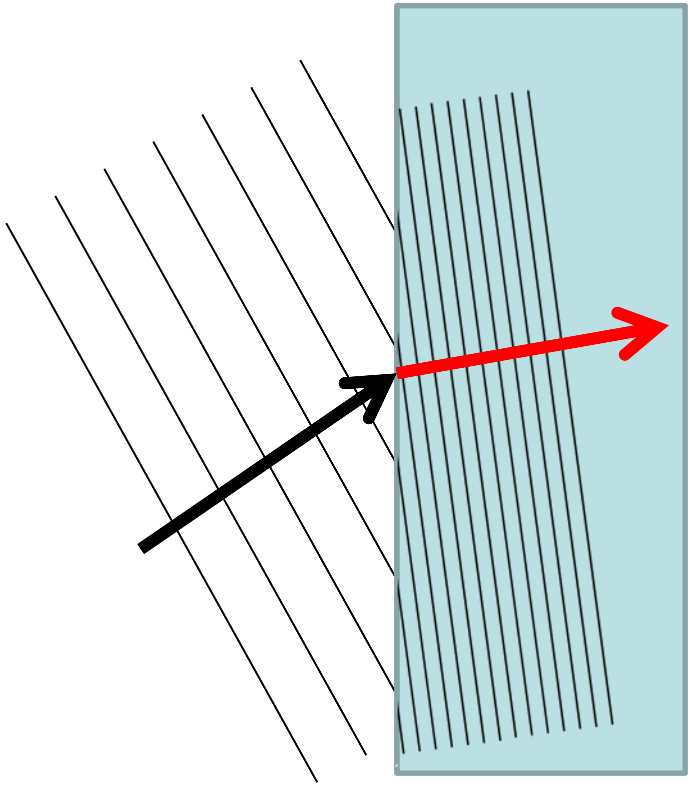


Generic solution steps:

Step 1: Whenever translation invariance: Use $k_{||}$ conservation
to find allowed refracted wave vectors

together with $$k_{||}^2 + k_z^2 = (n\omega/c)^2$$

Sketch of k|| conservation



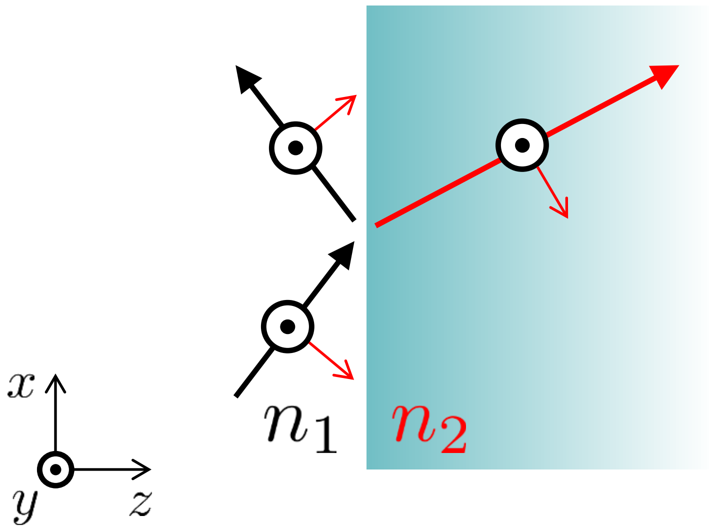
$k_{||}$ conservation:

The only way for the phase fronts to match

everywhere, any time

on the interface

Amplitude s-polarization



$$\mathbf{E}_1 = \mathbf{E}^y_{\rm in} e^{i\mathbf{k}_{||}x}[e^{+ik_{z1}z} + r e^{-ik_{z1}z}]$$

$$\mathbf{E}_2 = \mathbf{E}^y_{\rm in} e^{i\mathbf{k}_{||}x} t e^{+ik_{z2}z}$$

Remember $\mathbf{B} = \frac{1}{\omega}\mathbf{k} \times \mathbf{E}$

$$\mathbf{E}_{||} - \text{conservation}[E_y]: \quad (1+r) = t$$

$$\mathbf{H}_{||} - \text{conservation}[H_x]: \quad \mu_2 k_1 \cos\theta_1 (1-r) = \mu_1 t k_2 \cos\theta_2$$

Now eliminate *t* to obtain reflection coefficient *r (equal μ)*

“s-pol [senkrecht] – E perpendicular to plane of k’s”
p-pol - swap E for H

What you see from this problem

Scattering: incident field (plane wave) is split by object $\varepsilon(r)$

Translation invariance provides parallel momentum conservation

Boundary conditions determine everything to do with amplitude

Total internal reflection: if wave vector is too long to
be conserved across the interface

Exercise: total internal reflection still means evanescent field

Quiz

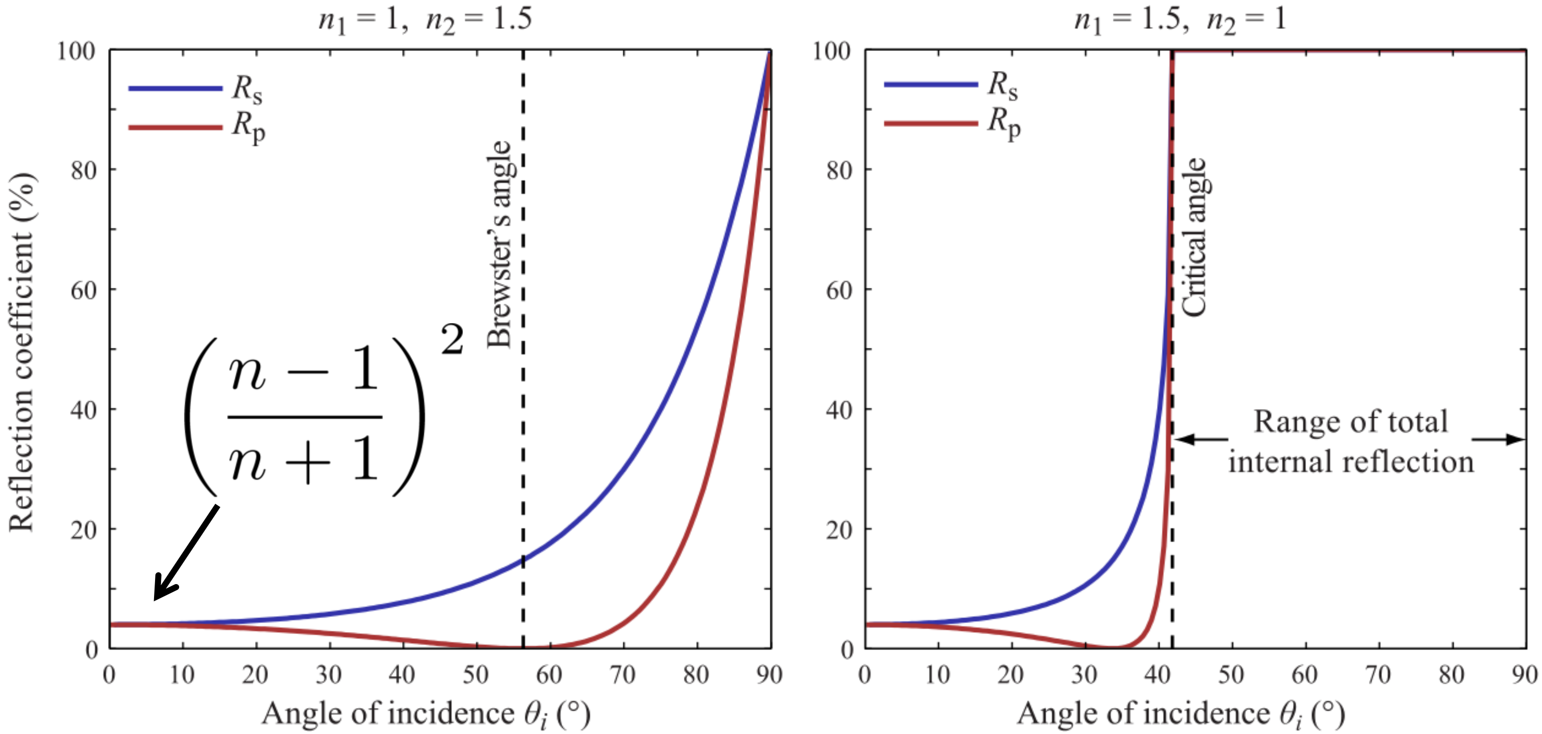
Silicon solar cells – does a wafer reflect more or less than 30%?

Are polarizing sunglasses vertically or horizontally polarized?

In total internal reflection is there **field** that sticks out into air?
A. 20 nm B. 200 nm C. 2 microns?

Is there a nonzero Poynting flux above the air interface?

Fresnel reflection



Plot from: Wikimedia Commons

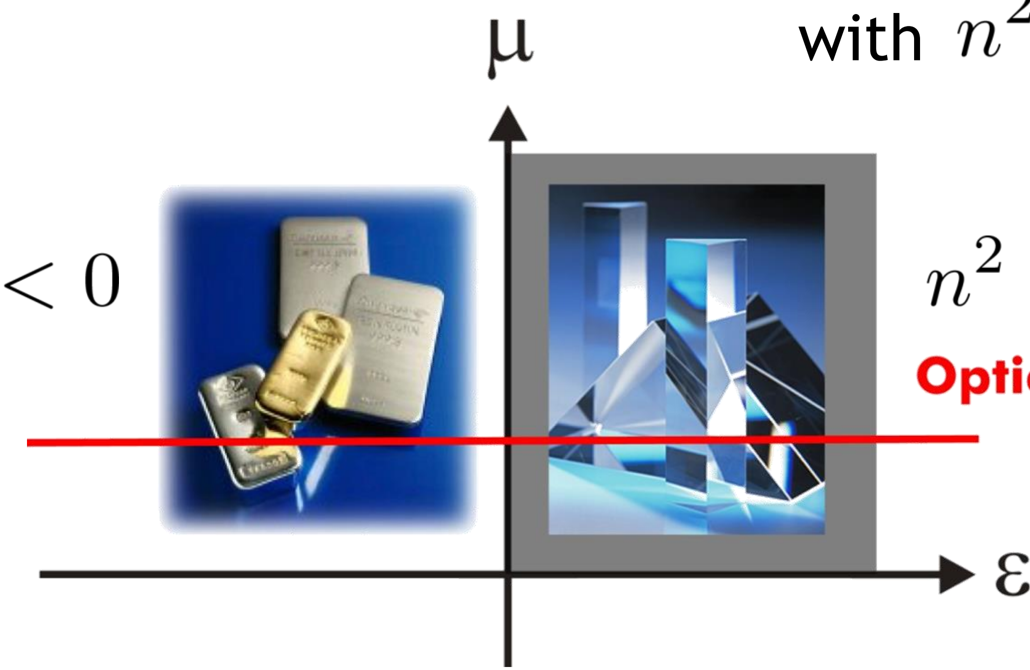
What n's does nature give us ?
Why ?

Optical materials

Plane waves of speed c/n

with $n^2 = \epsilon\mu$

$n^2 = \epsilon\mu < 0$



$n^2 = \epsilon\mu > 0$

Optical frequencies:
$\mu = 1$

Metals: reflective

Insulators: transparent

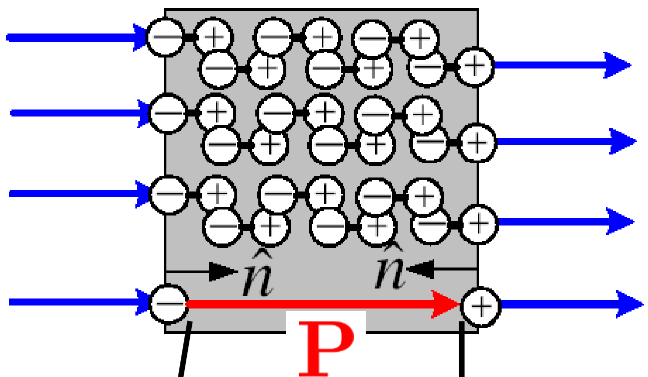
Dielectrics

Dielectric materials:

All charges are attached to specific atoms or molecules

Response to an electric field $\mathbf{E}$:

Microscopic displacement of charges



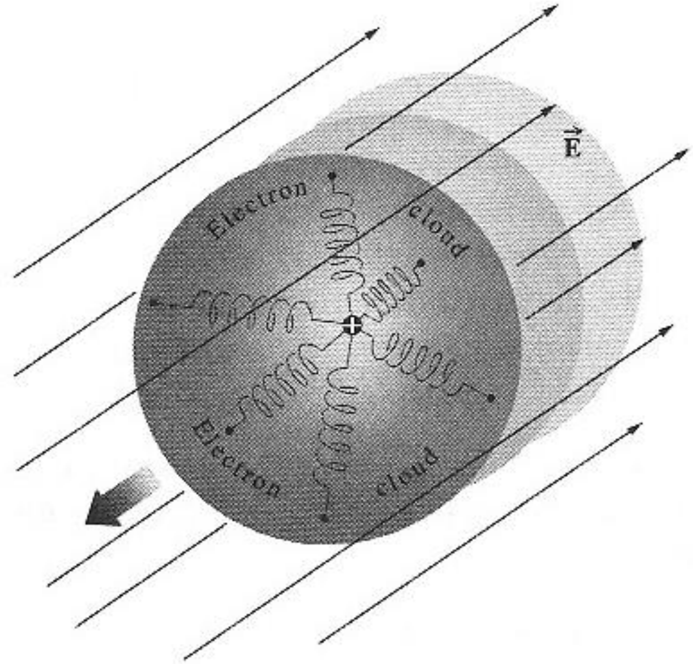
$$\mathbf{P} = \epsilon_0 \chi \mathbf{E} = \epsilon_0 (\epsilon - 1) \mathbf{E}$$

Macroscopic material properties: electric susceptibility χ, dielectric constant (or relative dielectric permittivity) ϵ

Atomic polarization

Equation of motion of electron:

$$m\frac{d^2\mathbf{x}}{dt^2} + m\gamma\frac{d\mathbf{x}}{dt} + k\mathbf{x} = -e\mathbf{E}$$



γ : damping coefficient for given material
k : restoring-force constant
resonance frequency $\omega_0 = \sqrt{k/m}$

Assume $\mathbf{E}$ is varying harmonically, and also $\mathbf{x} = \mathbf{x_0}e^{-i\omega t}$

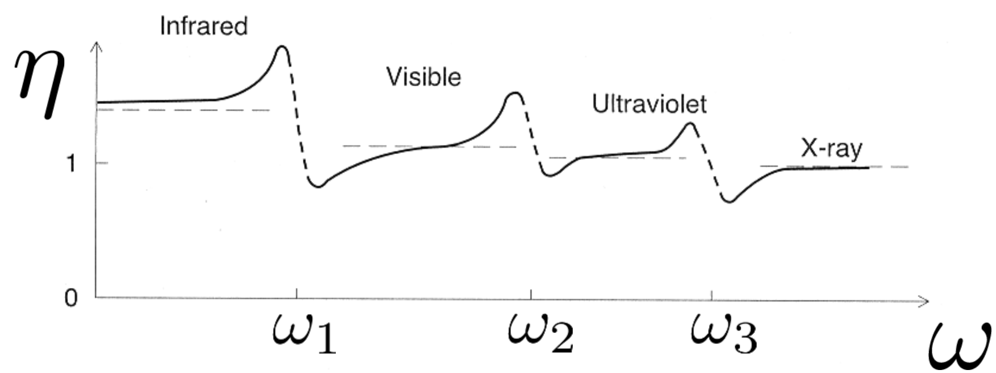
$$\Rightarrow \mathbf{P} = -Ne\mathbf{x} = \frac{Ne^2/m}{\omega_0^2 - \omega^2 - i\gamma\omega}\mathbf{E}$$

Typical solids

multiple resonances ω_j for Z electrons per molecule:

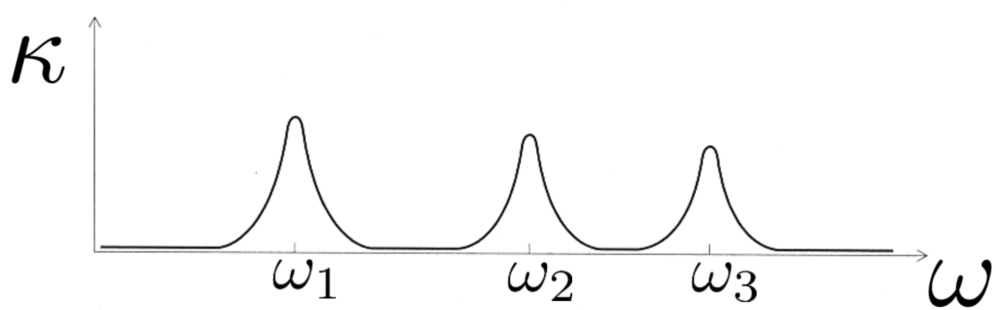
$$n^2 = 1 + \frac{Ne^2}{m\epsilon_0} \sum_j \frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j \omega} \qquad \sum_j f_j = Z$$

Where f_j is the oscillator strength or (quantum mechanically) the **transition probability**



n is a complex number:

$$n \equiv \frac{c}{\omega} k = \eta + i\kappa$$

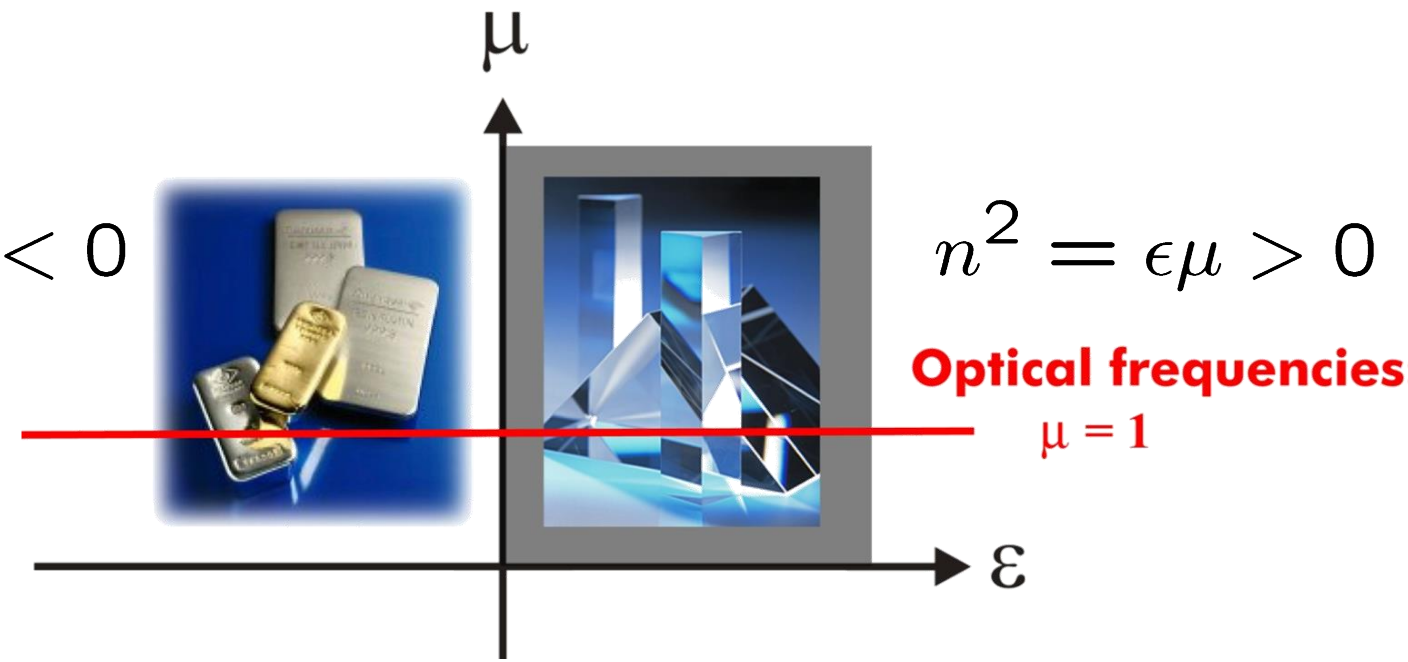


Optical materials

Optics deal with plane waves of speed c / n

with $n^2 = \varepsilon\mu$

$$n^2 = \epsilon\mu < 0$$



$$n^2 = \epsilon\mu > 0$$

Metals: reflective

Insulators: transparent

What do we know about metals ?

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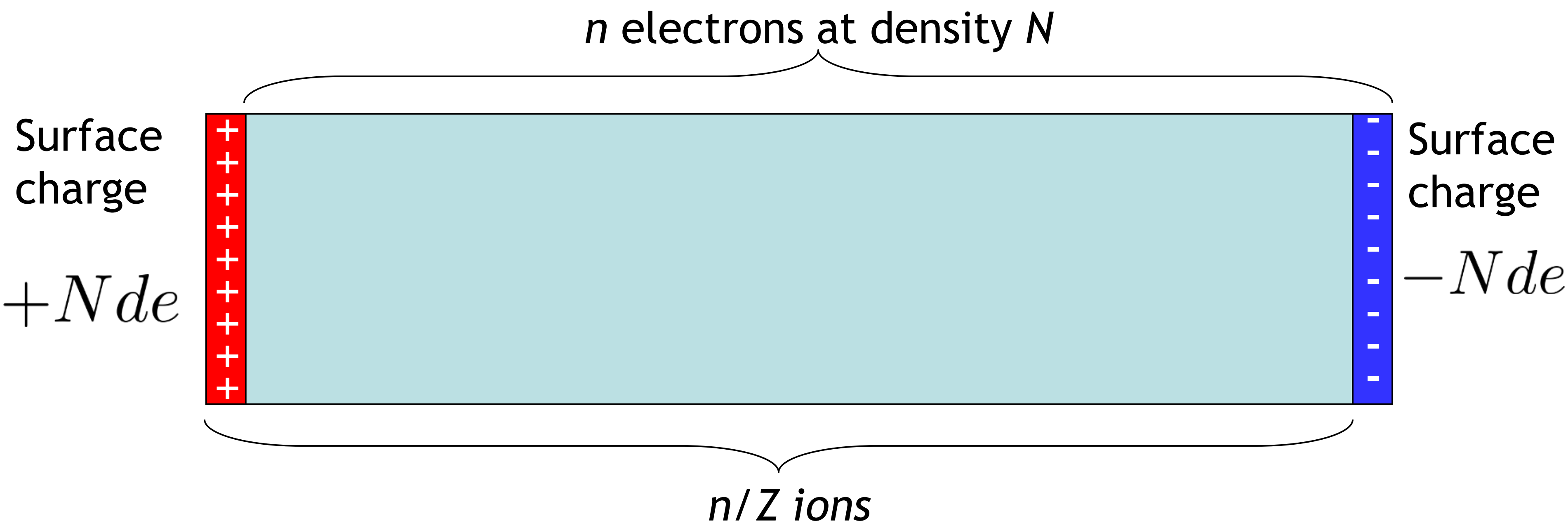
- Metals contain free electrons in an ionic backbone: ‘plasma’
- Static electric fields do not penetrate into metals. Fields are shielded by surface charges

Questions:

- What is the permittivity of a metal at high frequencies ?
- How quickly can charges move to shield fields ?

Plasma frequency – free charge oscillation

Suppose we displace the whole electron gas by a distance *d*



$$nm\ddot{d} = -ne\mathbf{E} = -ne(\sigma/\epsilon_0) = -nNe^2 d/\epsilon_0$$

Collective plasma oscillation: frequency $\omega_p = \sqrt{Ne^2/m\epsilon_0}$

Quick estimate

Silver: 1 electron per atom
Mass density: 10.5 g/cm^3
Atomic weight: 108 g/mole

$$\omega_p = \sqrt{\frac{Ne^2}{m\varepsilon_0}}$$

$N=5.8\ 10^{28}$ e- per m^3
$e=1.6\ 10^{-19}$ C
$m=9\ 10^{-31}$ kg
$\varepsilon_0=8.8\ 10^{-12}$ F/m

$\omega_p= 1.37\ 10^{16}\ s^{-1}$

$\lambda_p= 140$ nm

Actual value – renormalized by m* , effective mass of conduction electrons

Drude model

Drude model: conduction electrons with damping: equation of motion

$$\frac{d^2x}{d^2t} + \frac{1}{\tau}\frac{dx}{dt} = -\frac{eE}{m}e^{-i\omega t}$$

Free electrons: no restoring force

For the conductivity σ we find the 'Drude model':

$$j = -Ne\frac{dx}{dt} = \sigma E \qquad \sigma = \frac{\sigma_0}{1 - i\omega\tau} \quad \sigma_0 = \frac{Ne^2\tau}{m}$$

Drude model:
- The DC conductivity is set by the density of electrons
- The AC conductivity drops for frequencies approaching the electron relaxation rate

Converting conductivity to ε

Compare Maxwell in two forms (without/with currents):

$$\nabla \times H = \frac{\partial D}{\partial t} = \epsilon\epsilon_0 \frac{\partial E}{\partial t} \qquad \nabla \times H = \frac{\partial D}{\partial t} = j + \epsilon_0 \frac{\partial E}{\partial t}$$

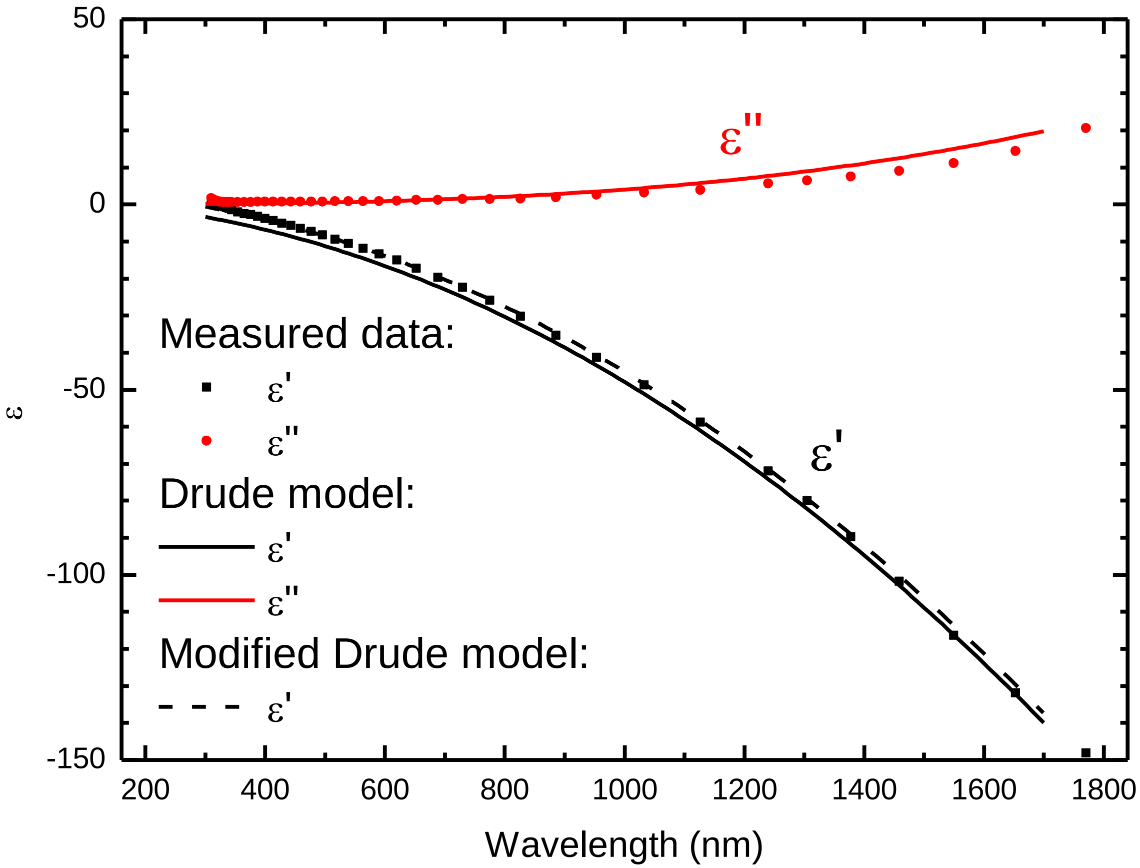
$$\epsilon = 1 + \frac{i\sigma}{\omega\epsilon_0} = 1 + \frac{i\sigma_0/\epsilon_0}{\omega(1 - i\omega\tau)} = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\gamma}$$

with collision rate $\gamma=1/\tau$ and plasma frequency

$$\omega_p = \sqrt{\frac{Ne^2}{m\epsilon_0}}$$

Special frequency scale depending on metal

Measured data and model for Ag



Drude model:

$$\varepsilon' \approx 1 - \frac{\omega_p^2}{\omega^2}, \quad \varepsilon'' \approx \frac{\omega_p^2}{\omega^3}\gamma$$

Modified Drude model:

$$\varepsilon' = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2}, \quad \varepsilon'' = \frac{\omega_p^2}{\omega^3}\gamma$$

Contribution of bound electrons

Ag: $\varepsilon_\infty = 3.4$

Take home message:

1) A "dielectric" has $\varepsilon > 1$, real
   Bound charges

2) A perfect metal has $\varepsilon = -\infty$, real
   perfect screening by free charges

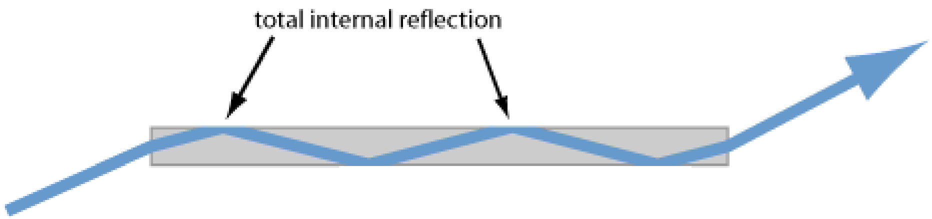
3) A real metal has Re $\varepsilon < 0$ up to the UV/visible

4) Im ε signifies *loss*. For metals this is *Ohmic resistance*

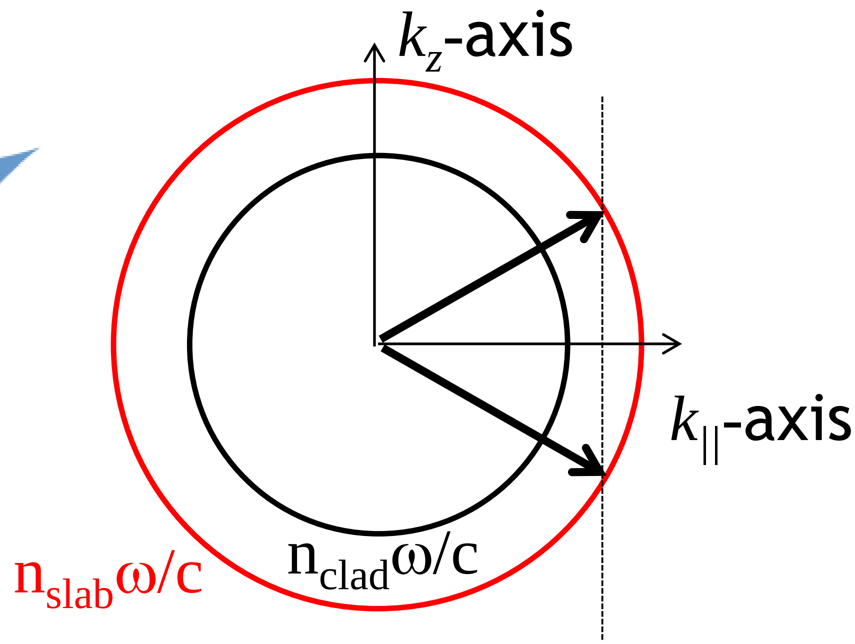
Are there any X-ray metallic mirrors ?

A doped semiconductor has free electrons..
does it look like a metal then?

Trapping light in waveguides



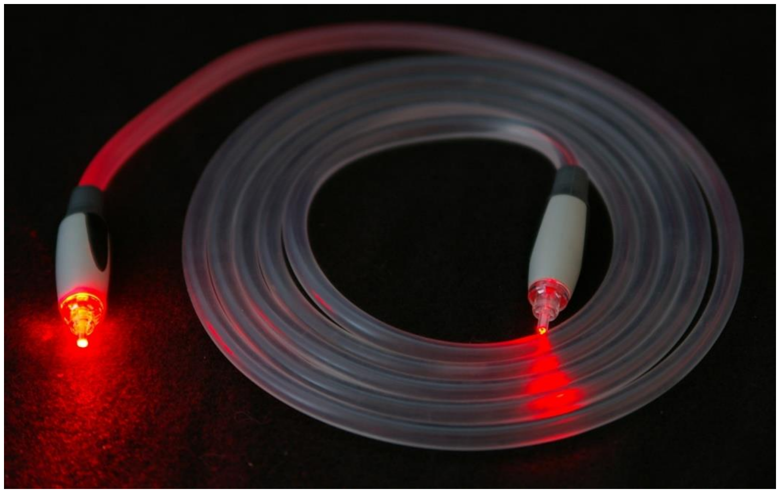
$$k_{||}^2 + k_z^2 = (n\omega/c)^2$$



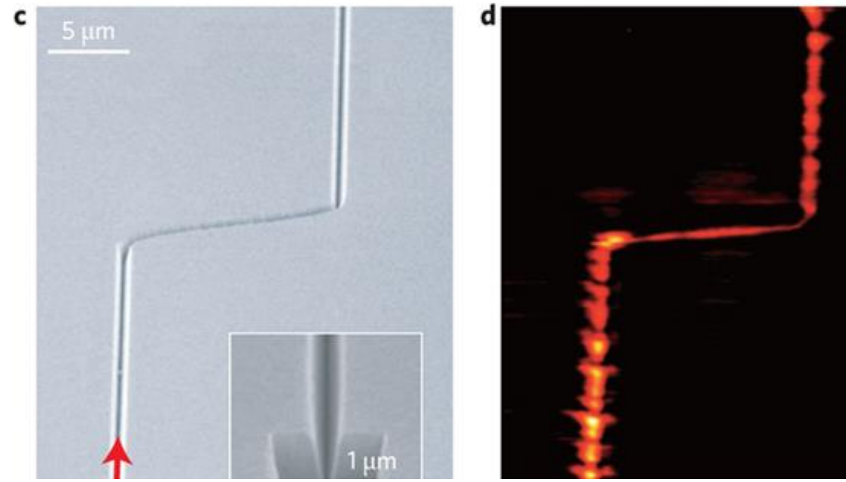
Guided mode if:

- In the *high* index medium $k_z^2>0$
- In the *low* index medium $k_z^2<0$ -*exponential tail*

Message



Wikimedia- optical fiber

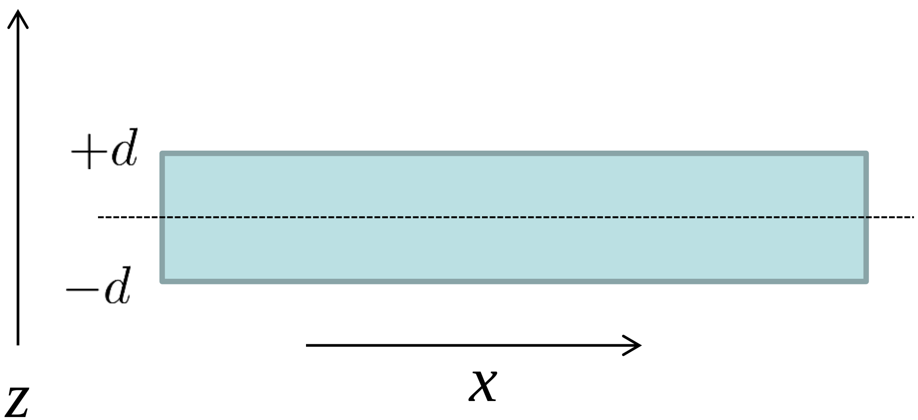


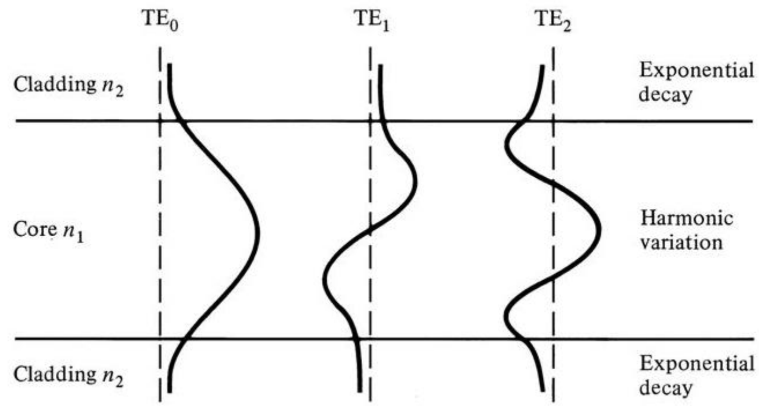
S. Bozhevolnyi, PRL **95** 046802

Dielectric ‘glass’ photonics
Guiding, but large modes > $(\lambda/2)^2$

Metal films, ridges, grooves
Modes as tight as $(\lambda_{vac}/100)^2$

Dielectric slab

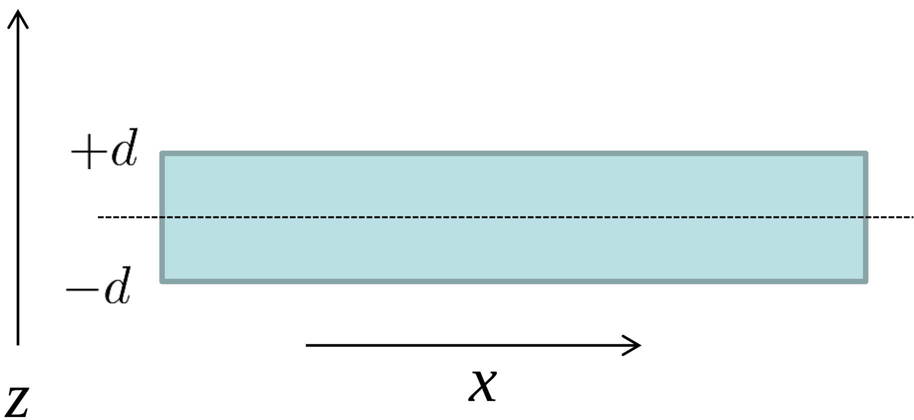




Exponential decay outside, oscillatory in the slab

Under which conditions do these exist?

Dielectric slab



$$H_{1,y} = H_1 e^{ik_{||}x - \kappa(z-d)}$$

$$H_{2,y} = H_{2\uparrow} e^{ik_{||}x + ik_{z2}z} + H_{2\downarrow} e^{ik_{||}x - ik_{z2}z}$$

$$H_{3,y} = H_3 e^{ik_{||}x + \kappa(z+d)}$$

Assumed exponential outside, oscillatory in the slab

Symmetric modes

$$H_1 = H_3 \quad H_{2\uparrow} = H_{2\downarrow}$$

Asymmetric modes

$$H_1 = -H_3 \quad H_{2\uparrow} = -H_{2\downarrow}$$

Dielectric slab

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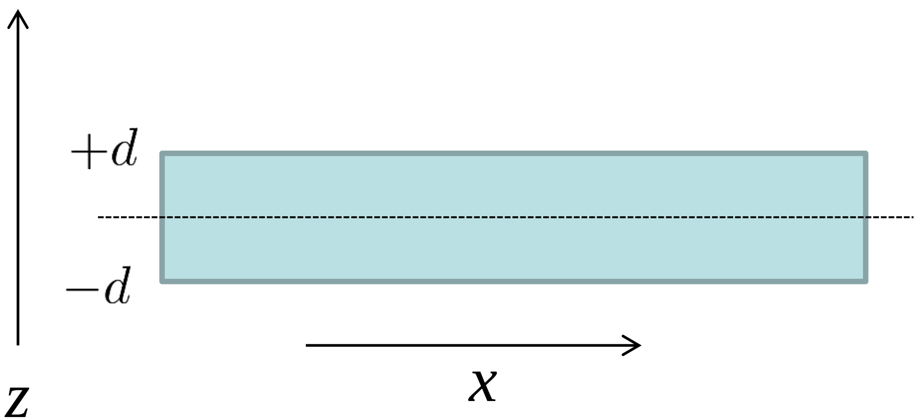

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$$H_{3,y} = H_3 e^{ik_{||}x + \kappa(z+d)}$$

Assumed exponential outside, oscillatory in the slab

Symmetric modes

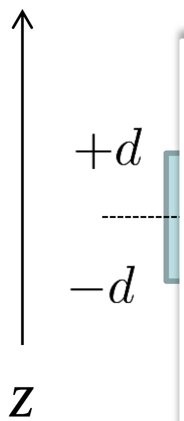
$$H_{1,y} = H_2(e^{ik_{z2}d} + e^{-ik_{z2}d})$$

$$\kappa H_{1,y} = \frac{ik_{z2}}{\epsilon} H_2(e^{ik_{z2}d} - e^{-ik_{z2}d})$$

Continous H

Continuous [k x H]$_{||}$= k$_z$H

Dielectric slab



$\downarrow e^{ik_{||}x - ik_{z2}z}$

Ass

Sy

Note the following:

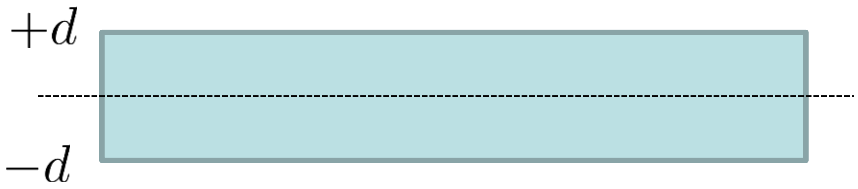
1. As before, we use $k_{||}$-conservation
2. We impose boundary conditions
3. Different from before, there is no *driving*

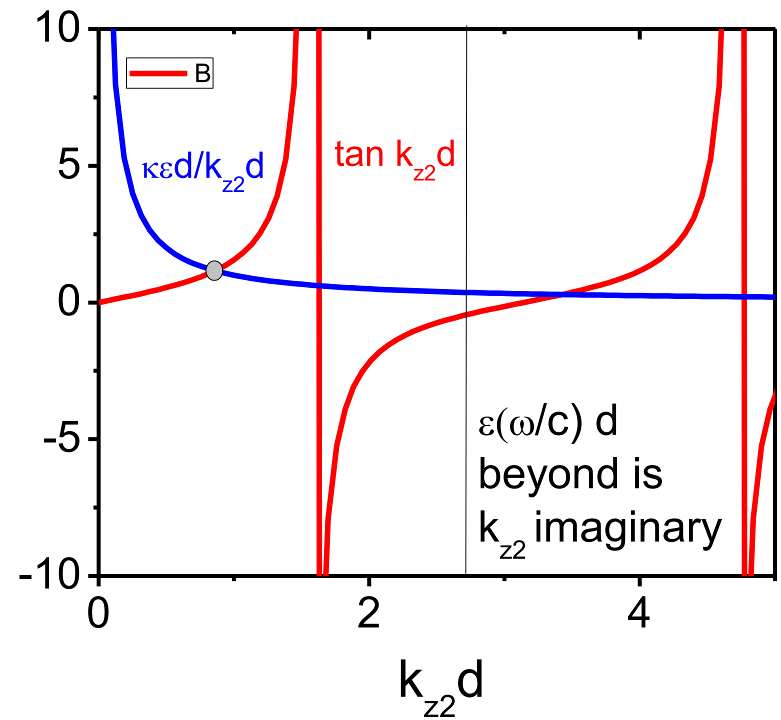
"Eigenmode exists with no driving"

H

$_{2\downarrow}$

Dielectric slab



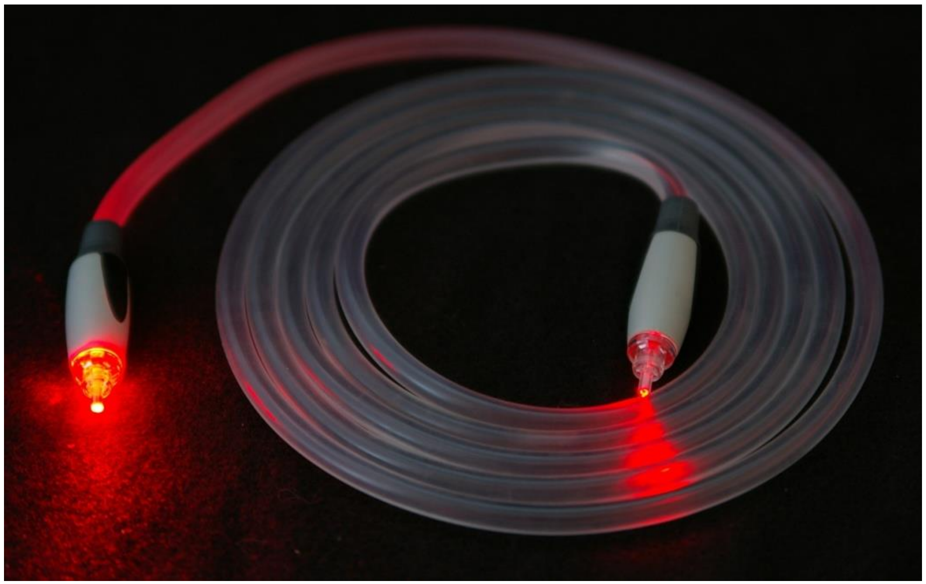


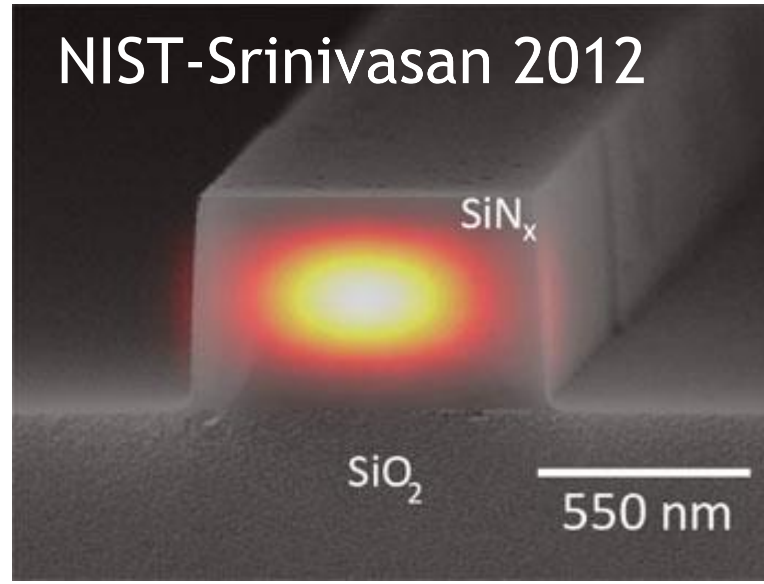
Symmetric modes

$$\tan k_{z2}d = \frac{\kappa\epsilon}{k_{z2}}$$

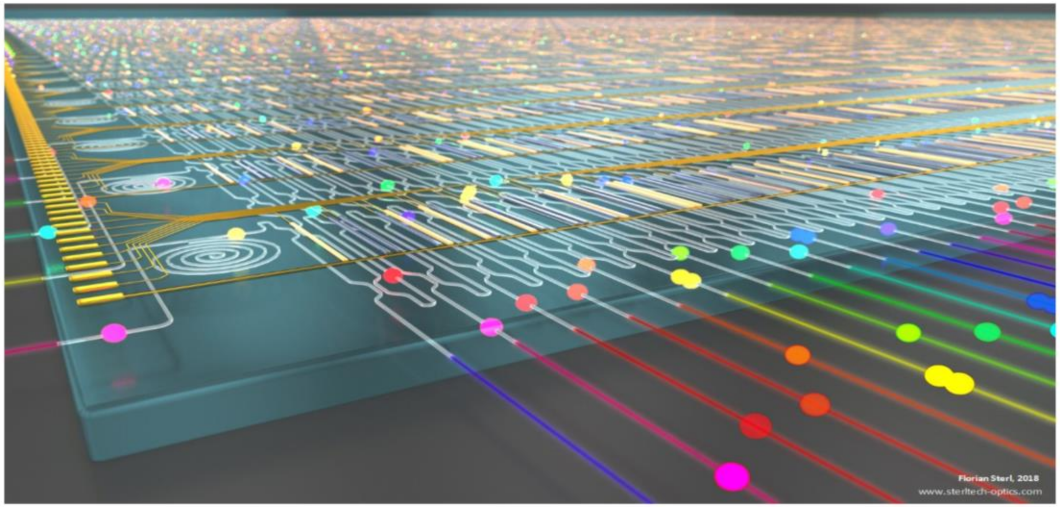
A symmetric dielectric 2D slab
has at least one symmetric guided mode for any thickness

Photonics with guided modes



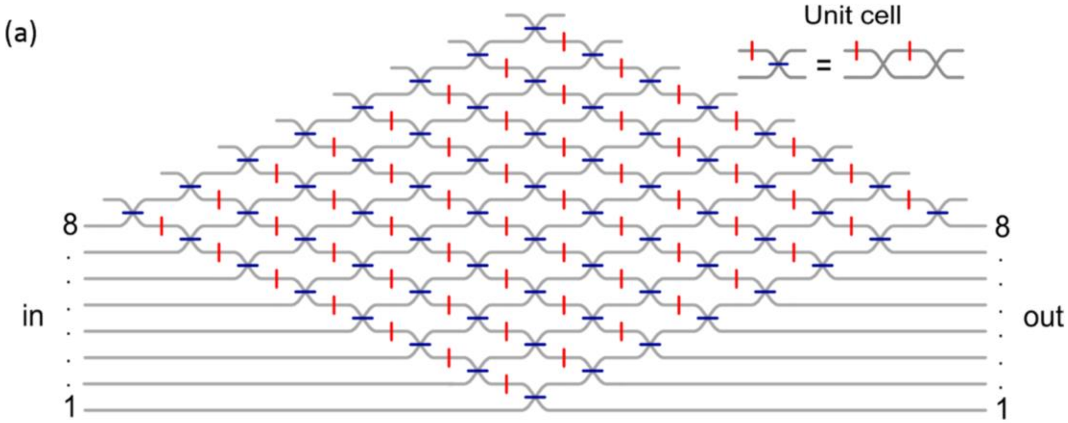


Confining light in 1 or 2D to guide it along a plane or line



ca. 5 cm sized chip
8x8 channels in & out

Each junction programmed



Indistinguishable
photon interferences

Quantum optics easier on
chip than in lab

Lionix, J. Renema, arxiv 1805:10999

Facts

Symmetric slab with lower index cladding always has at least one guided mode

Asymmetric geometries require minimum thickness and index

Generally, high refractive index contrast is required for good confinement

The mode is always at least as wide as the wavelength in the high n

Question

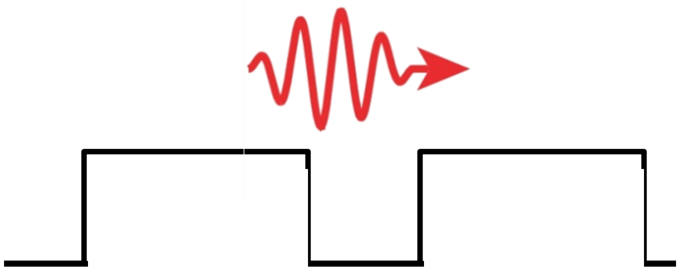
Isn’this just like “the hydrogen atom”?

A potential – refractive index - traps a photon?

Problem with trapping light

Scalar wave equation for light

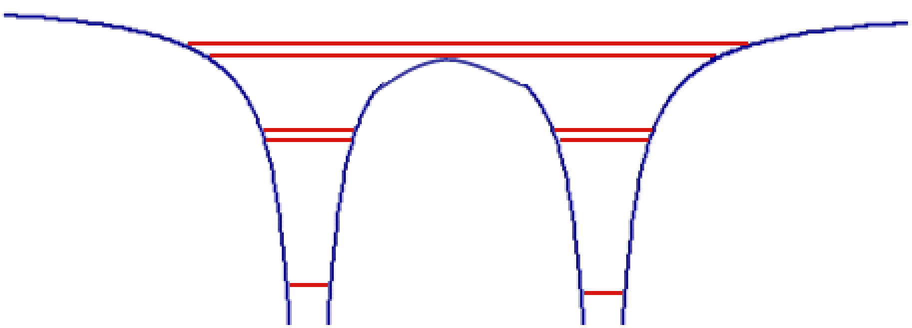
$$-\Delta\mathbf{E} + [1-\epsilon(\mathbf{r})]\frac{\omega^2}{c^2}\mathbf{E} = \frac{\omega^2}{c^2}\mathbf{E}$$



Schrödinger equation

$$-\Delta\psi + V(\vec{r})\psi = U\psi$$

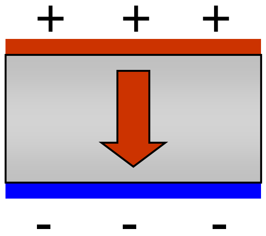
Confined states U < V



Light states are *always above the potential maximum in dielectrics*

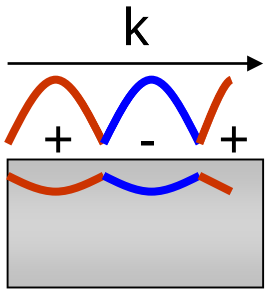
Trap requires either interference *or negative ε (metals $\Rightarrow$ plasmonics)*

From plasmon to plasmonics



Plasmons in the bulk oscillate at ω_p determined by the free electron density and effective mass

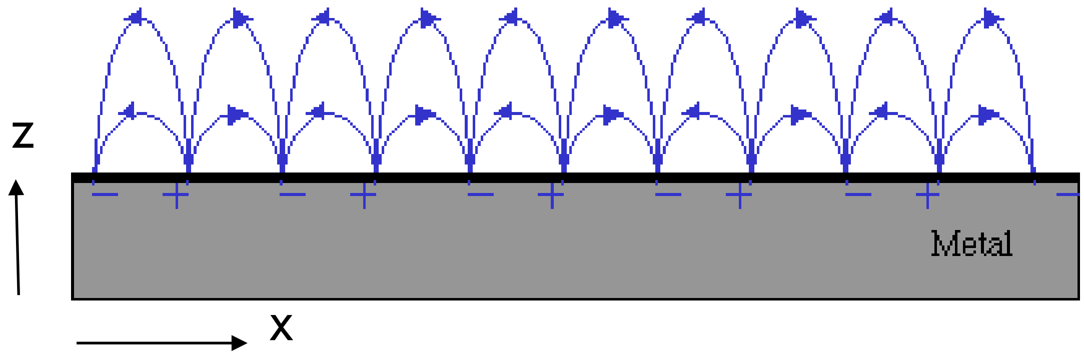
$$\omega_p = \sqrt{\frac{Ne^2}{m\epsilon_0}}$$

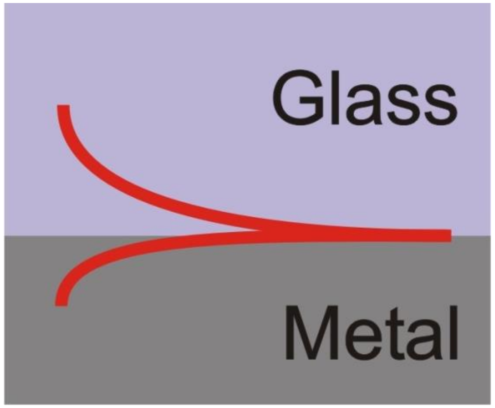


Plasmons confined to surfaces that can interact with light to form propagating "surface plasmon polaritons (SPP)"

$$\omega_{SPP} = \sqrt{\frac{1}{2}\frac{Ne^2}{m\epsilon_0}}$$

Surface plasmon polariton





$$\vec{E}_d(x,z,t) = \vec{E}_{d,0} e^{i(k_x x + k_z^d z - \omega t)}$$

$$\vec{E}_m(x,z,t) = \vec{E}_{m,0} e^{i(k_x x + k_z^m z - \omega t)}$$

For propagating bound waves:
- k_x is real
- k_z is imaginary

Polariton: light coupled to a material resonance
Plasmon polariton: EM wave coupled to plasma oscillations

Dispersion relation

Derivation of surface plasmon dispersion relation: k(ω)

We assume H is perpendicular to k and parallel to the surface
Note also the Ansatz *includes* parallel momentum conservation

$$\vec{H}_{d,m}(x,z,t,) = \begin{pmatrix} 0 \\ H_{y,d,m} \\ 0 \end{pmatrix} e^{i(k_x x + k_z^{d,m} - \omega t)}$$

Dispersion relation

Wave equation relates k_x, ε_{dm} and k_z in each medium:

$$(k_x^2 + k_{z,d,m}^2)H_{y,d,m} = \epsilon_{d,m}\frac{\omega^2}{c^2}H_{y,d,m}$$

Boundary conditions

$$H_{y,d} = H_{y,m}$$

Parallel H continuous across interface

Parallel E continuous

$$\vec{E}_{d,m} = \frac{1}{\epsilon_{d,m}\omega}\begin{pmatrix} k_x \\ 0 \\ k_z^{d,m} \end{pmatrix} \times \begin{pmatrix} 0 \\ H_y \\ 0 \end{pmatrix} = \frac{1}{\epsilon_{d,m}\omega}\begin{pmatrix} -k_z^{d,m}H_y \\ 0 \\ k_xH_y \end{pmatrix}$$

Circles

$$(k_x^2 + k_{z,d,m}^2) = \epsilon_{d,m} \frac{\omega^2}{c^2}$$

Dielectric: equation of a circle
Also: imaginary k_z for very large k_x

Metal: equation of a circle with an *imaginary radius*
simply means: *always exponentially confined in* some *direction*

For any real k_x, k_z is imaginary for $\varepsilon < 0$

Dispersion relation

Wave equation gives

$$(k_x^2 + k_{z,d}^2) = \epsilon_d \frac{\omega^2}{c^2}$$

$$(k_x^2 + k_{z,m}^2) = \epsilon_m \frac{\omega^2}{c^2}$$

Boundary condition yields

$$\frac{k_{z,d}}{\epsilon_d} = \frac{k_{z,m}}{\epsilon_m}$$

How to find a solution?

$$\frac{k_{z,d}^2}{k_{z,m}^2} = \frac{\epsilon_d^2}{\epsilon_m^2} = \frac{\epsilon_d \frac{\omega^2}{c^2} - k_x^2}{\epsilon_m \frac{\omega^2}{c^2} - k_x^2}$$

Dispersion relation

x-direction:

$$\boxed{k_x = k'_x + ik''_x = \frac{\omega}{c}\sqrt{\frac{\epsilon_m \epsilon_d}{\epsilon_m + \epsilon_d}}}$$

$$\boxed{\varepsilon(\omega)}$$

z-direction:

$$\boxed{k_{z,d} = k'_{z,d} + ik''_{z,d} = \frac{\omega}{c}\sqrt{\frac{\epsilon_d^2}{\epsilon_m + \epsilon_d}}}$$

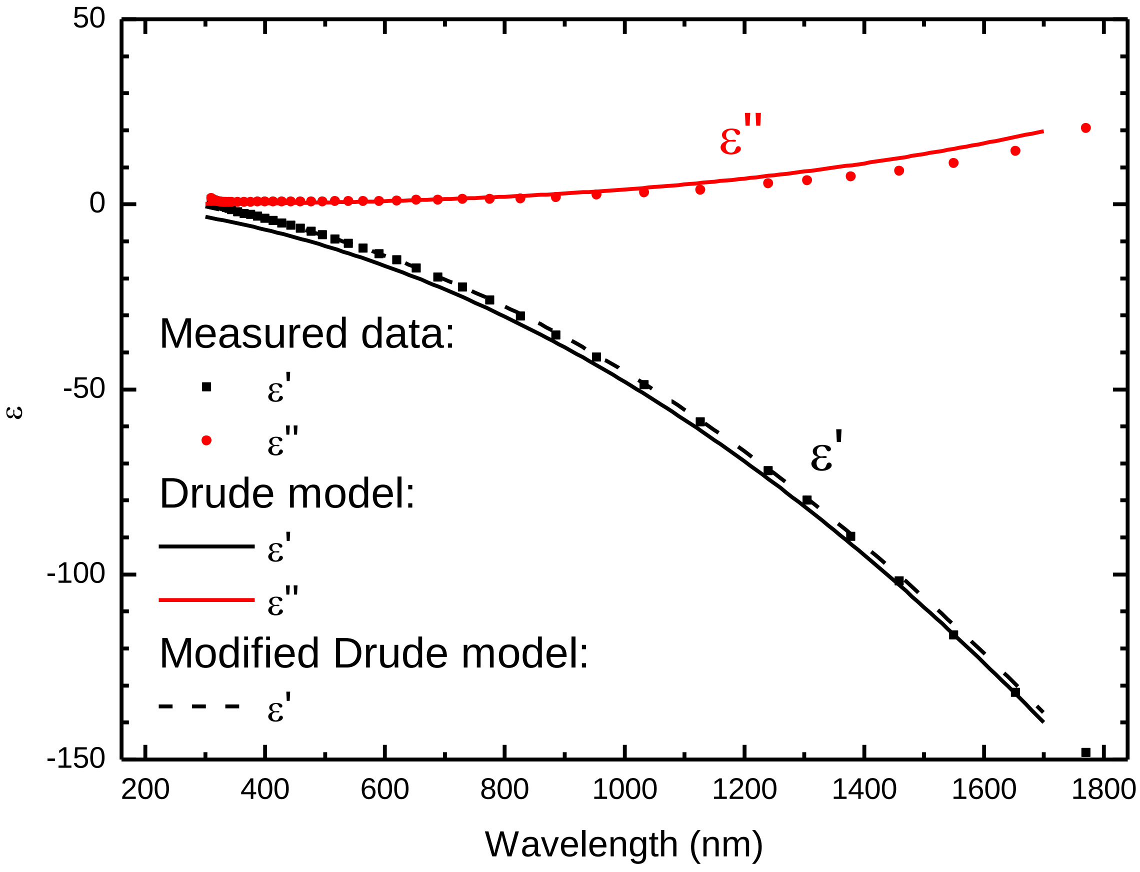
Bound SP mode exists for specific combinations of k, ω, and $\varepsilon(\omega)$

Requirement 1: k_z imaginary: $\varepsilon_m + \varepsilon_d < 0$,

Requirement 2: k_x real: $\varepsilon_m < 0$

Conclusion: $\varepsilon_m < -\varepsilon_d$

Measured data and model for Ag



Drude model:

$$\varepsilon' \approx 1 - \frac{\omega_p^2}{\omega^2}, \quad \varepsilon'' \approx \frac{\omega_p^2}{\omega^3}\gamma$$

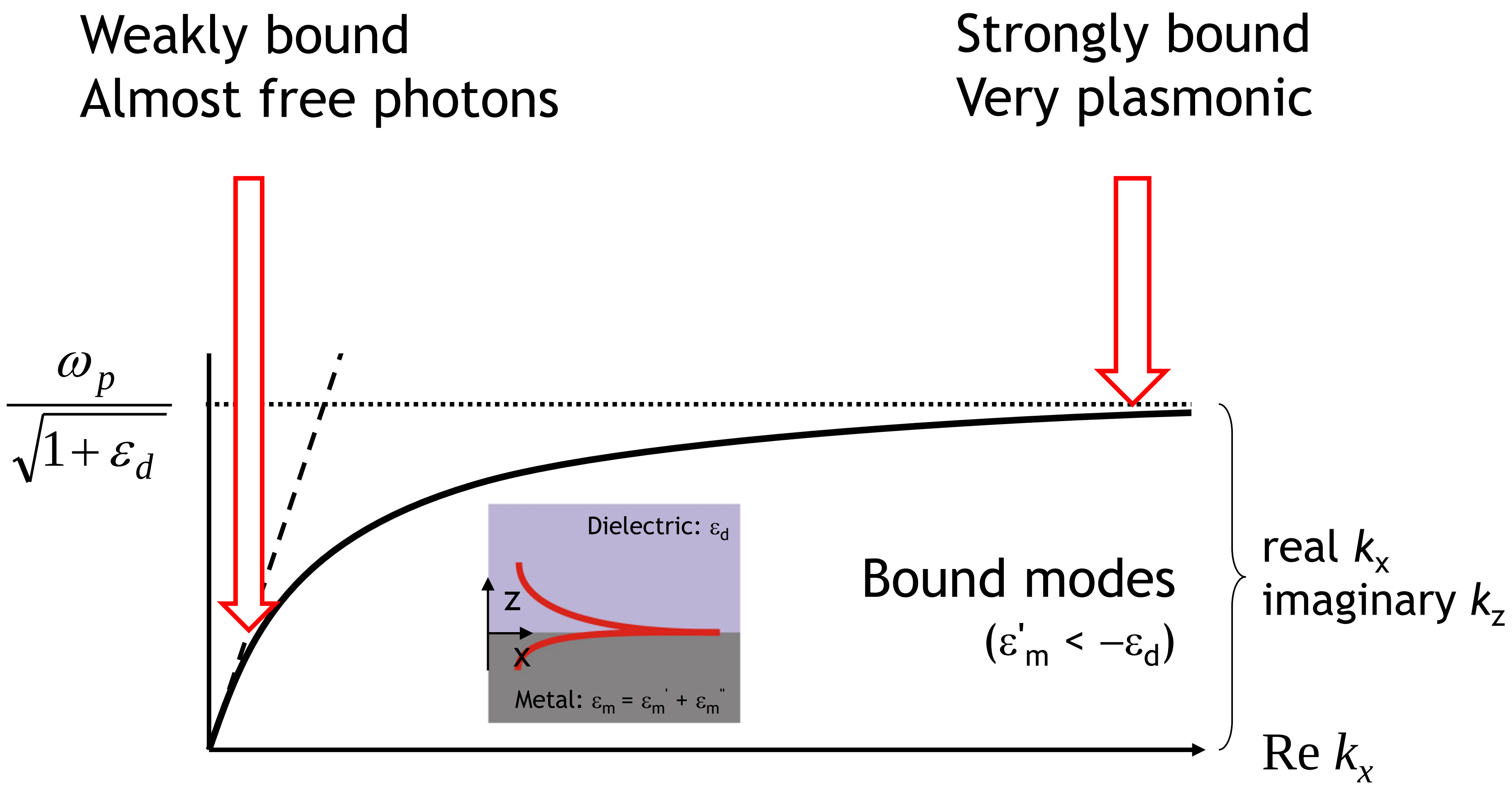
Modified Drude model:

$$\varepsilon' = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2}, \quad \varepsilon'' = \frac{\omega_p^2}{\omega^3}\gamma$$

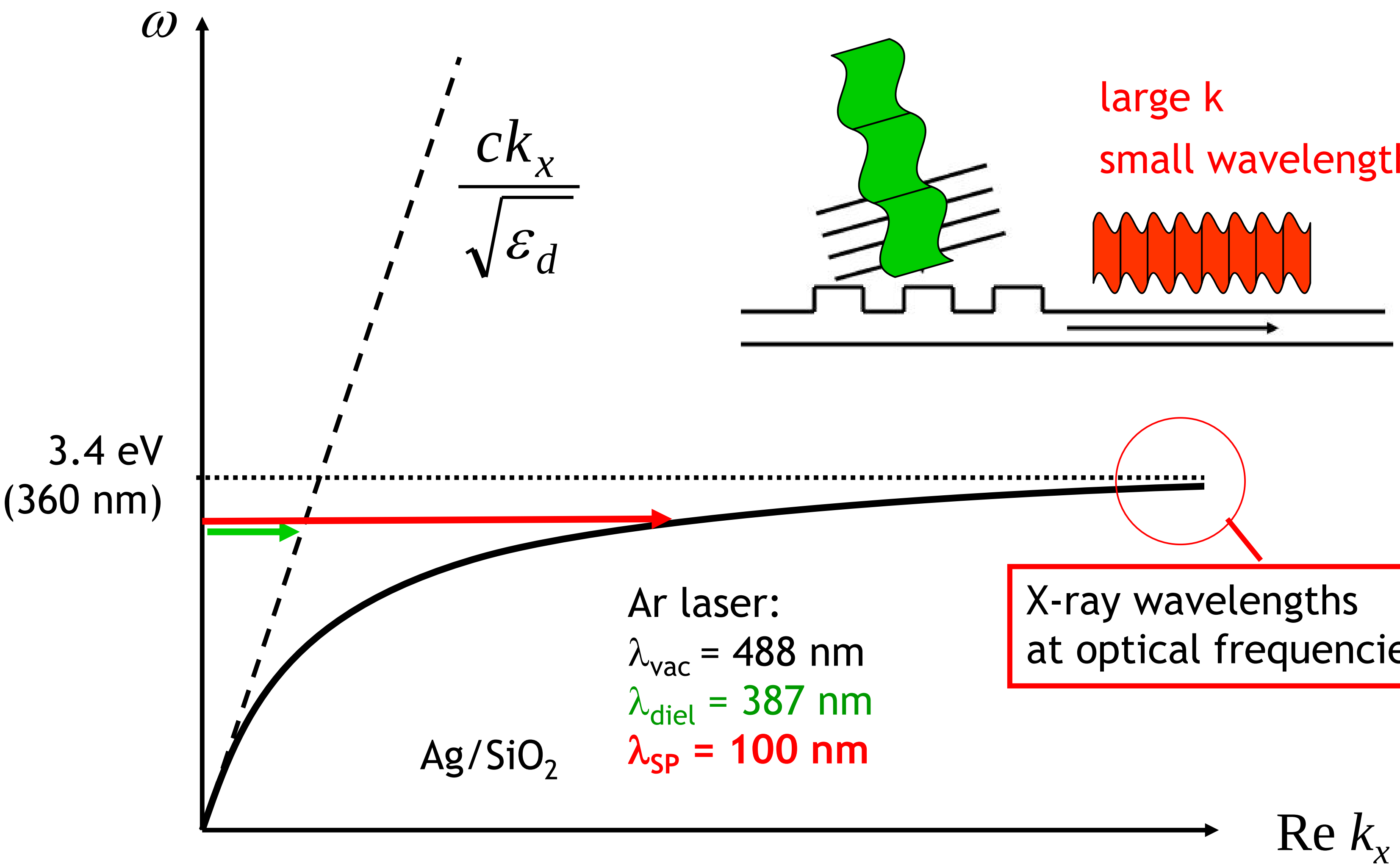
Contribution of bound electrons

Ag: $\varepsilon_\infty = 3.4$

Surface plasmon dispersion relation: $k_x = \frac{\omega}{c}\left(\frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d}\right)^{1/2}$



Surface plasmon dispersion relation: $k_x = \frac{\omega}{c}\left(\frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d}\right)^{1/2}$



Vector fields

1. Mode is truly transverse magnetic
2. Mode has strong longitudinal E-field
3. Mode has E-field normal to substrate

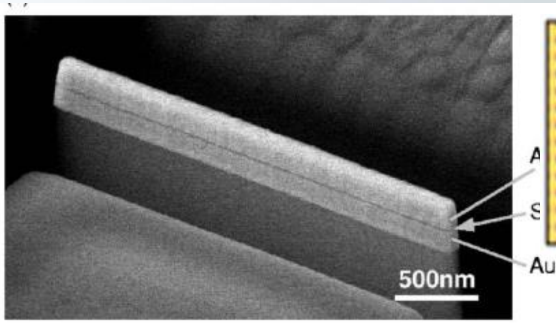
$$\vec{E}_{d,m} = \frac{1}{\epsilon_{d,m}\omega}\begin{pmatrix} -k_z^{d,m} H_y \\ 0 \\ k_x H_y \end{pmatrix}$$

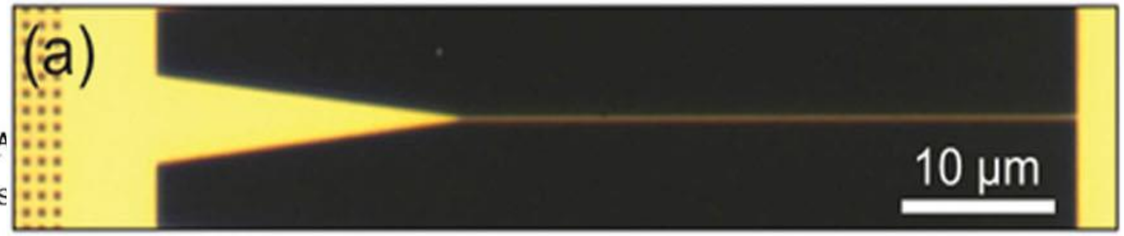
Note that $$k_{z,d}/k_x = \left|\frac{\epsilon_d}{\epsilon_m}\right|^{1/2}$$

Mode sticks out a distance ~ $\lambda/5$ or so from substrate

In the metal, the penetration depth is about 20 nm (skin depth)

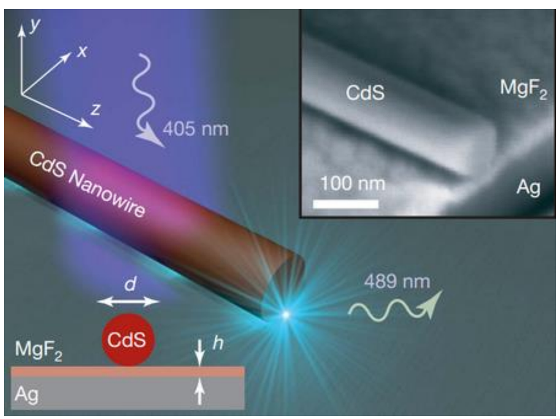
Flavours

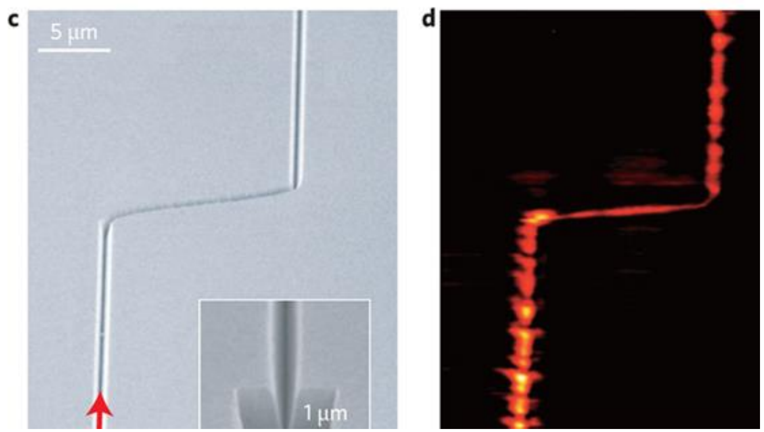


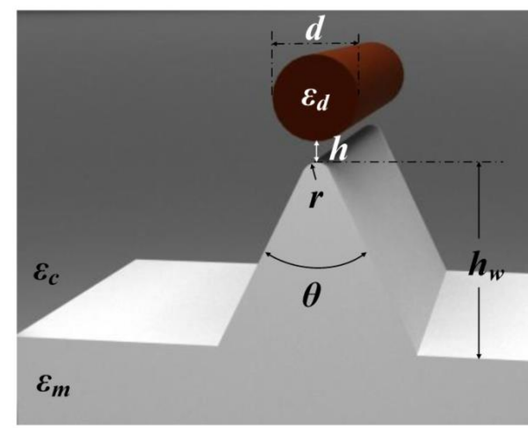


Taper and wire

M-I-M







Hybrid V-groove Wedge/hybrid

Modes as tight as $(\lambda_{vac}/100)^2$ cross section and $\lambda<\lambda_{vac}/50$

Kurokawa & Miyazaki [Tsukuba]
Oulton & Zhang [Berkeley]

Verhagen & Polman [Amsterdam]
Bozhevolnyi [SDK] / Bian [Beihang Univ]

Message

- Dielectric constant of isolators derives from *bound charges*
- $\varepsilon > 1$, generally below ~ 15, Lorentzian resonance staircase

- Metals have a plasma of free electrons
- The Drude model predicts deeply negative ε

- Dielectric high-index slabs and fibers can guide light
- Confinement is limited

- Plasmonics use the confinement of free electrons
- Extreme confinement
- Extreme dispersion – the cost is loss